John Farrar
Cambridge University

LECTURE

to the respects of
F.H.S.

ON THE

HISTORY OF MATHEMATICS

BY

FRANCIS H. SMITH, A. M.

SUPERINTENDENT AND PROFESSOR OF MATHEMATICS

OF THE

VIRGINIA MILITARY INSTITUTE.

[Published at the request of the Cadets.

"GAZETTE OFFICE," LEXINGTON, VA.
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CORRESPONDENCE.

V. M. INSTITUTE, JUNE 22, 1841.

SIR: We have been appointed a Committee on the part of the Corps of Cadets, to communicate to you the following resolution:

"*Resolved*, That we return our thanks to Major FRANCIS H. SMITH, for his able and satisfactory Lecture on the History of Mathematics, and that a Committee be appointed to request a copy for publication."

We beg leave to express the hope that you will comply with the request of the Cadets, and to assure you of the satisfaction which the publication of your Lecture would afford.

We have the honor to be,
Your obedient servants,

L. A. GARNETT,
J. W. BELL,
J. L. BRYAN,
} Committee.

Maj. F. H. SMITH.

V. M. INSTITUTE, JUNE 23, 1841.

GENTLEMEN: I beg leave to acknowledge the receipt of your polite note, communicating a resolution of the Corps of Cadets, requesting, for publication, a copy of my Lecture on the History of Mathematics.

I am deeply sensible of the flattering terms in which the Cadets have been pleased to express themselves in their Resolution, and it gives me great pleasure to comply with their request.

You must be aware, that my Lecture has been prepared under an unusual press of engagements, which has prevented me from entering as fully into detail as the nature and importance of the subject demanded; while I have not had it in my power to refer to many books, which would have added to the usefulness and accuracy of the Lecture. I present it, however, as it is, in the humble hope that it may in some measure promote the cause of Science in our beloved old Commonwealth.

With sentiments of high regard,
I am your friend and servant,
FRANCIS H. SMITH.

Cadets L. A. GARNETT,
J. W. BELL,
J. L. BRYAN,
} Committee.

LECTURE.

No occupation can be more agreeable to the student, than tracing the rise and progress of those subjects, which have claimed his time and attention. The inquisitive character of the human mind does not permit him to rest satisfied with the *results*, merely, of science or of art, but leads him to seek the causes of these results, and the chain of circumstances connected with their developement. The admirer of the beauties of Shakspeare, finds his delight increased, if any reminiscence can be discovered, throwing the least light on the subject or composition of a single play; and a new interest invests the recovery of a simple relict, illustrative of the character or habits of the Bard of Avon. The original copy-right of "Paradise Lost" was with difficulty sold for a few shillings, while the Antiquarian now readily gives a hundred pounds for the manuscript of the Blind Poet. With what increased pleasure do we take up the works of Sir Walter Scott, after reading the interesting biography of Lockhart? Each novel attracts a new feeling, each character a new interest, and we recommence their perusal with a relish never before experienced.

Nor are these enquiries without instruction, especially

when investigating subjects of a scientific character. The knowledge of the means by which certain results have been obtained, will always facilitate us in the progress of our studies, while we may be also led to the most important discoveries. It was a geometrical problem, solved in some particular cases by the ancients, which led Descartes to the discovery of the science of Analytical Geometry; and the Differential Calculus was suggested to Newton, by the consideration of a geometrical property, which was known as far back as the time of Euclid.

The importance given by our Laws to the study of the Mathematics, arising from their acknowledged value in every pursuit of life, has suggested to me the propriety of presenting to you a brief history of their rise and progress. This step is rendered the more necessary from the fact, that so little has been published on the subject; while the works that have appeared are for the most part inaccessible to you. These embarrassments have retarded in no small degree my own investigations; but I am unwilling to withhold from you, on this account, whatever may aid or interest you in the progress of your studies. My object being to present to you a *history* of the Mathematics, I shall not aim at any originality, but shall simply collect such facts, mentioned by writers on this subject, as may be useful for future reference. My authorities are *Montuela-Histoire des Mathematiques*, *Encyclopædia Metropolitana*, *Playfairs History of the physical sciences*, *Hallam's Literature of Europe* and *Professor Leslie's Arithmetic.*

The Science of Arithmetic is so nearly co-existent with the exercise of our mental faculties, that it is impossible to trace with accuracy the steps of its early introduction. A child at the earliest age acquires the habit of comparing quantities with one another, and the result of this comparison will gradually lead him to the idea of *number*. Beyond this simple conception, his progress will be slow; for until words are formed, he will be unable to separate the idea of any number, from the qualities of the objects with which it is associated. He will have

a distinct idea of *four* cows, as distinguished from *five* cows, but the idea of these numbers, will not necessarily be the same, as those which represent a like number of sheep. If *words*, however, be used to represent these numbers, which shall be independent of any qualities of the objects with which they were at first associated, he will soon become accustomed to the words, without any reference to such associations. This process for the formation of abstract numbers might be thus completely effected, by attaching names to the series of natural numbers. Our progress in numeration would still be exceedingly limited, if the names thus assigned were arbitrary, and entirely independent of each other. The memory could not retain such a multitude of disconnected words, while the operations of Arithmetic would be more difficult than could be mastered in the infancy of society. Upon this hypothesis, we might readily conceive the mode of formation of the decimal system of numeration. A person in the habit of counting on his fingers would be led to divide numbers into classes, the units in each class increasing in a tenfold proportion. Names being given to the first nine digits, and also to the units in each ascending class, he might, by combining the names of the digits with those of the units of *local* value, express any number by the simplest composition of words. This classification leads to a nomenclature which is simple and comprehensive. An examination of the systems of numerical language of various nations shows that it has been established upon such a principle as is here supposed. The decimal system of numeration will be found to have met with very general adoption, a fact which can only be accounted for from the natural practice of numbering by the fingers on the two hands.

The Decimal scale will not, however, be found to be the only scale used by the ancients. The habit of counting upon the toes as well as the fingers, would naturally suggest a *vicenary* scale, which is to be traced among some of the nations of antiquity. Again, a person may stop after counting the fingers on one hand; or he may

count the fingers on both hands and repeat those on one, by the first combination would be formed the *quinary* scale, and by the latter, the *denary* with the quinary scale subordinate. The use of *two* hands in separating objects into *pairs*, and the prevalence of *binary* combinations in the human body, would also naturally, lead to another scale of numeration. All of these scales being formed upon natural combinations, are the only systems of numeration which have ever met with *general* adoption.

Of all the systems of numerical words that of Thibet, possesses the most simple structure, and makes the nearest approach to arithmetical notation of local value. The first twenty-nine numerals are

1 Cheic	11 Chucheic	21 Gnea-cheic
2 Gnea	12 Chugnea	22 Gnea-gnea
3 Soom	13 Chusum	23 Gnea-soom
4 Zea	14 Chuzea	24 Gnea-zea
5 Gna	15 Chugna	25 Gnea-gna
6 Tree	16 Chutree	26 Gnea-tree
7 Toon	17 Chutoon	27 Gnea-toon
8 Ghe	18 Chughe	28 Gnea-ghe
9 Goo	19 Chugoo	29 Gnea-goo
10 Chutumbha	20 Gnea-chutumbha	

Here the numerals from ten to nine, are formed by the combination of the first syllable of the word for *ten*, with the names of the first nine numbers; but from twenty-one to twenty-nine, the name for *two*, *gnea*, acquires a value from *position*, in a manner which bears the closest analogy to our present system of notation.

The Chinese possess a very extensive and perfect system of numeration. In alphabetical languages, there is no connexion between numerical words, and numerical symbols, the latter being always of subsequent invention to the former. But the Chinese numerals being either simple elements, or composed of them, like other characters, are transformed to the oral language, upon the same principles, by which monosyllabic sounds are attached to all their characters. They use three kinds of

symbolical representatives. The first is used in historical and scientific works, the second in bonds and formal instruments, and the third for mercantile purposes. Other characters are conventionally used in legal instruments, to which other meanings are attached. Thus, the character used for *one* means perfection, that for *two* a word which means to *assist*, for three an *accusation*.

As perfect as the Chinese system of notation is, they are not in possession of the system by 9 figures, and zero, and hence have no claims to this great invention.

Before proceeding to an account of the symbolical method of numeration, it may be proper to notice a species of digital arithmetic, which was much practised by the ancients. It consists in denoting the nine digits, and the articulate numbers as far as 100, by inflections of the *left* hand, whilst the hundreds were marked on the *right* hand, by similar inflections, which were used to denote the articulate numbers on the left; and the thousands were a repetition on the *right* hand of the inflections used for digits. They were thus enabled to denote all numbers, that were less than 10,000. The "venerable Bede" afterwards extended this method of numeration to embrace a million. A knowledge of this species of numeration is necessary, to understand many passages in classical writers. Juvenal states it as peculiar felicity of Nestor, that he counted the years of his age on his *right* hand.

> Felix nimirum, qui tot per sæcula mortem
> Distulit, atque suos jam *dextra* computat annos.
>
> Sat X. 248.

The image of Janus was represented according to Pliny, with his fingers so placed, that they noted 365, the number of days in the year.

With respect to the different methods of symbolical arithmetic, we will commence with the Greek. They expressed the natural numbers below 10,000 by means of the letters of the alphabet and three other symbols, which denoted 6, 90, 900. The following table

exhibits the four classes of digits of the first, second and third order.

1.	α	β	γ	δ	ε	ϛ	ζ	η	θ
	1	2	3	4	5	6	7	8	9
2.	ι	κ	λ	μ	ν	ξ	ο	π	ϟ
	10	20	30	40	50	60	70	80	90
3.	ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
	100	200	300	400	500	600	700	800	900
4.	α	β	γ	ζ	ε	&c.	&c.		
	ͺ	ͺ	ͺ	ͺ	ͺ				
	1000	2000	3000	4000	5000				

The 4th class is a repetition of the first with an iota subscribed which augmented the number a thousand fold. The limit of Greek notation as far as depended on the preceding table, was only 9999. Their language, however, contained a term μυριας for the next superior unit, so that their numeration by words exceeded that by symbols. By making use of the letter M subscribed or postscribed to the symbols for any number in the above table, their values were augmented a thousand fold. In this manner the arithmetical notation of the Greeks was made co-extensive with the powers of expression of their numerical language, embracing eight places of figures. This notation subsequently underwent great improvements by the aid of the suggestions of the Archimedes, Apollonius and Diophantus.

There are no means of arriving at the Greek arithmetical operations, and they can only be imperfectly ascertained from the works of Archimedes and others in which these operations enter. In forming squares they commenced at the right and proceeded to the left, each product being written down separately. Fractions were variously represented. Most commonly when the denominator is 1, the numerator was placed on the right of the denominator and was marked by an accent. The introduction of the sexigesimal arithmetic by Ptolemy, was a great improvement in the science of calculation. The division of the circle into 360 degrees seems to have been suggested to the early astronomers by this number being nearly equal to the number of days in the year. From the convenience arising from its having a great number

of divisions, this division was adhered to, in the more improved state of the science. The sexigesimal division was not confined to the circle. The side of the inscribed Hexagon, which is equal to Radius, was itself divided into 600 parts, and again into 600 inferior parts. This enabled astronomers to avoid fractions, the operations of which by their ordinary arithmetic were exceedingly embarrassing. The sexigesimal notation as employed by Ptolemy was nearly the same as that used at present. The degrees were considered as units, and written in the usual manner, a stroke being placed over the last symbol, thus $\mu\bar{\varsigma}$ or 44°. The successive orders of sexigesimals, primes and seconds were denoted by one two and three accents. By this system all symbols beyond 60 were superfluous. The sexigesimal division of the circle has continued to the present day. An effort was made in France to substitute the decimal division, but without success, although recommended by the high authority of La Place. The French use the Decimal division, but its general introduction would have rendered useless all the existing tables, which were calculatcd with much labor to the sexigesimal division. The sexigesimal division of Time also continues. This division of the Radius continued until 1464 when Regiomontanus divided it into 10,000,000 parts.

The Arithmetical notation of the Romans was exceedingly cumbrous and imperfect. They made use of the letters of the alphabet to represent the first numerals, and by combining them in various and irregular methods, were able to express numbers of great magnitude. The Romans were very indifferent to scientific improvement, and one striking evidence of this indifference appears from the fact, that their tedious system of notation was replaced by the more perfect and comprehensive notation of the Greeks. The notation of the Romans was adopted in every part of their extensive empire, and continued to be employed wherever the Latin language was used, long after the introduction of the Arabic numerals, from a feeling of respect for antiquity, and a desire of conform-

ing in every particular to classical authors. The structure of the Roman arithmetical notation will account for the total absence of all arithmetical operations amongst them, which were not performed mechanically.

The ancient *Arabic* characters were derived from the Syriac, and were only 22 in number. The modern characters were introduced about the year 800, and are 28 in number.

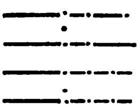
The invention of the notation by nine figures and zero, has given rise to many controversies. The total revolution which it effected in the systems of calculation makes it an important circumstance in the history of this science. The Hindoos possessed a very perfect idea of arithmetic at a very early period. Two works on Arithmetic are now in existence, written by Bhasçara, which enjoyed a high reputation in Hindoostan. As well as can be ascertained, Bhascara is supposed to have lived about the middle of the 12th century. Brahmegupta another writer on arithmetic, who is quoted by Bhascara, and parts of whose works are still extant, is supposed to have lived as early as the 7th century. And again, Arya-bhatta, who is referred to by Brahmegupta, and who is regarded as the oldest of the Hindoo uninspired writers, flourished as early as the 5th century. The original invention of the notation by nine figures and zero, is no where referred to, by any of these writers, but is always regarded as of Divine origin. It must therefore have preceded all existing records. The testimony of the Arabs is decisive as to the source from which they derived it. The first Arabian who wrote upon Algebra and Arithmetic, was Mohammed ben Musa, who lived towards the end of the 9th century. In this work reference is made to the Indian mode of computation, and the practice of subsequent Arabian writers, of writing those figures from the left to the right, after the manner of the Hindoos, but which is contrary to their own order, confirms the idea that the Hindoos possessed this mode of notation before the Persians, Arabs or any western people. This notation was introduced into Spain by the

Moors about the year 1136, into Italy in 1200, and shortly afterwards was carried into England by English travellers through Spain.

The operations of Arithmetic as practised by the ancients were mechanical. The Romans used a machine called the *Abacus*. This machine was a table in which were arranged grooves of different lengths. The numbers were denoted by small counters which moved in these parallel grooves. There were 7 divisions for whole numbers, representing the different orders of units as high as millions, the value of each superior unit, being denoted by the symbol which is placed between the long and the short grooves. The counters in the long grooves represented units, those in the short, five. Thus to denote 6, one counter would be placed in the long groove corresponding to simple units, and one in the short groove. To express 60, one counter would be placed in the long groove of the tens, and one in the corresponding short groove. As late as the 17th century arithmetical calculations were made entirely with counters. Shakspeare in his "Winters Tale" (Act 14, sc. 3,) represents the clown as embarrassed by an example in arithmetic, and declaring he could not do it without *counters*. In early times it was customary for merchants, Bankers and Judges to appear on a *bench* or *bank*, with a table before them on which their calculations were made with counters. The name of Court of Exchequer is derived from scaccarium, a quadrangular board, around which the officers or judges were seated. It was covered with cloth, divided by lines at right angles to each other, and small coins were used for counters, by means of which they adjusted the accounts of all litigants.

This system of palpable arithmetic has met with some advocates even in the present day. Professor Leslie regards it as the best means of giving to students philosophical ideas of the construction and classification of numbers. Let any number of parallel lines be drawn, representing different units. A counter placed on the lower line

represents 1, one on the second line 10, one between two lines is equivalent to 5 units of the next lower line. The number 4738 would be represented as in the figure.



Many systems of palpable arithmetic have been devised to facilitate these calculations, and the subject has been clothed with importance from the fact, that by these machines in a great measure, Arithmetic is taught to the Blind. Saunderson's calculating board was arranged with this view. Its inventor was blind at an early age, but by these simple contrivances became a distinguished mathematician. The Board used by him was perforated and pins of different sized heads were used to represent diffeient numbers.

Although the nomenclature of the ancients made their arithmetical operations slow, we owe to them many of the most important discoveries on the properties of numbers. The 7th 8th, 9th and 10th books of Euclid form an abstract of the most rational part of the science of arithmetic as known to the ancients, divested by the rigid form of his demonstration of the absurdities and refinement of the ancient schools,

The ancients had a great passion for tracing the mystical properties of numbers. Petrus Bungus wrote a volume of 700 quarto pages, on the mathematical, metaphysical and theological properties of numbers. He refers to every passage in the Bible where a number is mentioned and dilates particularly upon the number of 666, the number of the Beast, in Revelations, the symbol of Antichrist, and labors to show that this number represents the name of *Martin Luther*. Lucus de Burgo, the author of the first printed book on arithmetic, enlarges upon the number three. There are *three* principal sins, *avarice*, *luxury* and *pride*, there are *three* orders in the ministry—*Bishops*, *Priests* and *Deacons*, &c.

One of the most curious specimens of the ancient arithmetical writings, is the work of Bhascara. It is styled the Lilavati. This was the name of Bhascara's

daughter, whom he would not allow to marry, in consequence of having discovered by an astrological scheme, that such an event would have been fatal to his own life. It was by way of consolation, that he dedicated to her this work, and called it by her name. The following example in Multiplication will show the character of the book:

"Beautiful and dear Lilavati, whose eyes are like a fawn's, tell me what are the numbers resulting from one hundred and thirty-five, taken into 12? If thou be skilled in multiplication, by whole or parts, whether by division or separation of digits, tell me, auspicious woman, what is the quotient of the products, divided by the same multiplier?"

After the introduction of the Arabic numerals, the science of arithmetic very rapidly rose to a very elevated position. The operations were simplified and improvements made in all its departments. The fundamental operations of arithmetic as sketched in the Lilavati were eight in number, addition, subtraction, multiplication, division, square, square root, cube and cube root. In addition the Hindoos commenced indifferently on the right or left; each addition being regarded separately. Our present mode of subtraction is due to Garth, an English mathematician. The Lilavati contains no less than *six* different modes of multiplication, showing that this was regarded as a difficult operation by the ancients. The Hindoos and Arabians were ignorant of the present method. It was invented by Pythagoras, and was introduced into Europe by the Italians. We owe Decimal Fractions to the labors of Archimedes, Stifelius and Stevinus. Lord Napier subsequently added to their suggestions the use of the comma to separate the whole number from the fraction.

It will be observed that those magnitudes have been adopted as primary units of length, whose dimensions were so small as to be in some measure indivisible, frequent reference being had to the parts of the human body. Thus in Italy, *piccoli* is an expression of extreme minute-

ness. Among the Greeks, we find the δακτυλος, a *finger* πους, a *foot*. The Romans had the same term *digitus* and also *pollex*, a *thumb's breadth*, or *inch*. The English use three *barley corns* for an *inch*, taken from the middle of the ear and put end to end. For longer measure, where the parts of the human body could not be conveniently used, less uniformity appears in the selection of the units. *Poles*, *rods*, &c. were taken from the instruments used in land surveying. In the East distances were measured by *hours* or *day's journey*. In the days of *archery*, the *bow-shot* was a measure of space, and we now have the term *gun shot*. The term *furlong* comes from the furrow's length—as if furrow long. The *mille* or *mille passus* of the Romans, is the origin of the term *mile*, varying from its extreme length in the German mile of 22,500 feet, to the Italian of 5,000 feet; which shows that a classical name was borrowed, to express a *long* distance, without any regard to the precise amount. The term league comes from the German *lugen*, to see, and expresses the distance that can be seen on a plain with the naked eye. Measures of weight are mostly of an arbitrary character. The grain is often taken as the basis among the Hindoos, Greeks and Arabians. The English use *three pounds*, that for Troy and Apothecaries weight, being the same, but differently divided. The subdivisions of the *avoirdupois* weight correspond with those of the apothecaries. The term *Troy* is said to have been derived from the town *Troyes*, where a celebrated fair was held, and where this weight was used. The *avoirdupois* was introduced by statute of Henry the 8th, fixing the maximum prices of provisions in times of scarcity, and that they should be sold for the lawful weight, called *haber de pois*.

The origin of *Geometry*, like that of the other ancient sciences is involved in obscurity. Its derivation from two Greek words γη *earth*, and μετρεω *measure*, shows clearly that its principal application in the early ages of the world, was the measurement and division of lands.

Whether this be its true origin or not, it furnishes a reason for its cultivation.

The first traces of this science are found in Egypt, whence it was transplanted into Greece by the celebrated philosopher Thales. This distinguished sage lived about 640 A. C. and being unable to satisfy his thirst for knowledge in his native country, travelled into Egypt, and there became acquainted with the learning of that country. The genius of the Greeks is peculiarly exemplified in their geometry. *Euclid* stands without a rival, and so perfect is his work as to draw forth the well merited compliment of Lord Bacon, that "nothing had been added to the science since his day worthy of the lapse of so many centuries." He connected the elementary truths of Geometry into one great chain, beginning with the axioms, and extending to the properties of the five Regular Solids. Archimedes assailed the more difficult problems, and by means of the method of exhaustions demonstrated many curious problems, relative to the lengths and areas of curves, and the contents of solids. *Apollonius* treated conic sections, and the beautiful contrivance of the Geometrical analysis is very generally ascribed to *Plato*.

To *Thales* who is the first Grecian Geometer, many important discoveries are attributed. He is said to have been the first to show that the angles in a semi-circle are right angles, and he also demonstrated many important properties of triangles. *Pythagorus* is the next Grecian geometer of any note. He lived 500 A. C. and like Thales obtained his knowledge of this science from Egypt. To this philosopher we are indebted for the important proposition which forms the 47th of Euclid, that the square on the hypothenuse of a right angled triangle, is equivalent to the sum of the squares on the other two sides; and also that of all plane bodies, the circle has the greatest area under a given circumference.

Plato lived 390 years before Christ, and after travelling through Egypt established a school in Greece, over which he placed the celebrated inscription, "Let no man enter

here who is ignorant of Geometry." He considered this as the first of all human sciences, and although he left no express work on the subject, there is no doubt he was very profound in it.

The science of Geometry continued to flourish and to assume great importance under these celebrated masters and their disciples, when a great addition was made to their number in the celebrated *Euclid.* Euclid flourished under the 1st of the Ptolemies, about 280 years before Christ. His place of birth is not certainly known, though it appears he studied at Athens before settling in Alexandria. He is represented by his Commentator, Pappus, as possessing an excellent moral character, gentle and modest, and particularly kind to those who studied the mathematical sciences. His elements have had a great many commentators. Theon was the first who commented upon them early in the 4th century. Afier Theon, the sciences underwent the persecution and afterwards the patronage of the Arabs, and to an Arabian version of the Elements, we are indebted for the first Latin editions by Athelard, and Campanus in Italy, about the 12th or 13th century. The Greek text appeared for the first time at Basle, in 1553, edited by Simon Gnynæus, and this has been the foundation of the various editions which have since been published. Simson's edition was from this, and appeared in 1756.

The year 250 A. C. introduced us to the Prince of Mathematicians, *Archimedes.* He was the first who discovered an approximate ratio between the diameter and the circumference of a circle. The ingenious process by which he arrived at this result, is the same as that which is demonstrated in modern works on Geometry. He first employed polygons of six sides, and then by continual bisection found the perimeters of the polygons, which differed by an inconceivably small quantity from that of the inscribed and circumscribed circle. He thus found that the diameter was to the circumference, as 7 was to some number between 21 and 22. To Archimedes we also owe the determination of the ratio between a sphere

and the circumscribed cylinder, and many of the most important properties of the conic sections. His reasoning is a model of accuracy, but it is prolix and difficult. The works of Archimedes are the most precious relict of antiquity.

Next in fame to Archimedes stands *Apollonius*; who flourished 240 years before the Christian Era. He was surnamed by his cotemporaries the *Great Geometer*. He composed many valuable works on the science, most of which are lost or remain only in scattered fragments. His treatise on Conic Sections is nearly entire, and fully justifies the high reputation he had acquired.

Few distinguished geometers are found after Apollonius among the Greeks, and this period must be regarded as the time when Greece passed the zenith of her scientific fame. Science in general, was indeed fast declining, and the only names of distinction betwen this time and the fall of Alexandria are Pappus, Theon, Diocles and Proclus. *Pappus* flourished 380 A. D. He published a book of Mathematical collections, in which he endeavored to collect the scattered discoveries, and illustrate the writings, of the ancient Mathematicians. This book is valuable on account of the connected history it gives of early geometrical knowledge. The original Greek has never been published. The only translation by Commandine appeared in 1558. *Theon* is principally distinguished for his commentaries on Euclid. He was the father of the accomplished *Hypatia*, who distinguished herself to such a degree by the cultivation of the mathematical sciences, that she was chosen as the successor to her father in the Alexandrian school, where she became illustrious by her talents and virtues. She was sacrificed by a fanatical mob, in the 5th century. *Proclus* the chief of the Platonists at Athens, signalized himself by his commentaries on Euclid, and *Diocles* was principally distinguished as the author of the *Cissoid* curve, which in honor of him was called the *Cissoid of Diocles*. After these, few names appear among the mathematicians of this period, possessing any interest. All who had

made science their study, were at the beginning of the 7th century at the celebrated school of Alexandria, where the light of the Grecian science was finally to be extinguished. The successors of Mahommed in the political and religious controversy which distinguished this period, devastated the whole of the vast extent of territory which stretches from the east to the Southern limits of Europe. All who cultivated the arts and sciences, and who had sought an abode in Alexandria, either for the purposes of pursuing their studies, or of taking refuge from the cruelty of these bigoted contestants, were expelled with ignominy or inhumanly butchered by their conquerors. The valuable library which was deposited in the museum, and which contained the scientific labors of so many centuries, was totally destroyed; the Caliph Omar in giving orders for its destruction, observing, "that if any of its works agreed with the Koran, they were useless, and if they did not, they ought to be destroyed." This happened in the year 640, of the Christian era. A few of the Cultivators of science escaped the fury of their persecutors, and by the aid of the manuscripts taken with them in their flight, were enabled to prosecute their studies in the countries in which they found refuge. Destitute as they were, they would have been able to do but little, had not the Arabians in less than two centuries after this catastrophe, become the cultivators of those very sciences, to whose destruction they had contributed so largely. By their diligence and perseverance in studying the old Greek manuscripts, the translations and works of all the Grecian geometers, whose writings are preserved, have come down to us. The works of Archimedes and Apollonius have thus been preserved and transmitted.

The *Romans* were at no time distinguished for their knowledge of the exact sciences. They studied Astronomy, but not so much from love of the science itself, as from a desire to cultivate astrology and pry into the mysteries of the future. Few writers appear among these of any consequence, and with the exception of *Boetius*

and *Vitruvius*, the Romans can boast of no names of standing in this Department of knowledge. Vitruvius published a work on Architecture which is quoted at the present day. From the destruction of the Alexandrian library for a period of nearly 600 years, we find but little done towards the promotion of science. This was truly the dark age for science, as it was for every thing else. The "*venerable Bede*" who lived 700 A. D. and *Roger Bacon* in 1240 A. D. are the only individuals, who during this long period displayed any mathematical skill, but no discoveries are due to them. In the 13th century, the cause of letters began to revive, and we find the names of *John de Sacro Bosco*, or *John of Halifax*, who wrote a treatise on the sphere, and *Campanus* of Navarre, who translated Euclid, and wrote on the quadrature of the Circle, among the geometers of this period. The poet Chaucer was also a distinguished mathematician in the 15th century. *Purbach* lived in the 14th century. *Muller* or *Regiomontanus*, *Lucus de Burgo* and *Copernicus*, added much to the geometrical knowledge, by their commentaries and works. *Purbach* corrected by the Greek text the ancient version of Archimedes, and translated the Conics of Appollonius. *Regiomontanus* substituted for the sexigesimal division of the Radius, that of its being composed of 10,000,000 of parts, and he calculated new tables for every degree and minute of the quadrant. *Lucus de Burgo* revived Campanus' translation of Euclid, while *Copernicus* directed his labors chiefly to the science of Astronomy. In the 15th century *Commandine* published translations of several of the Greek geometers, among which is particularly distinguished the mathematical collections of Pappus.

The name of *Vieta* stands pre-eminent among the mathematicians of this period. He was a Frenchman, and lived 1540 A D. He was to a degree the inventor of literal algebra, and his improvements in this science will be presently noticed. He was equally distinguished as a Geometer, and his treatise on Trigonometry con-

tains many practical problems on the quadrature of the Circle and duplicature of the Cube. Many illustrious geometers appear in the 17th century; among whom may be mentioned *Lucas Valerius*, an Italian, who determined the centres of gravity of the Conoid, spheroid, and their segments;—*Snellius*, a dutchman, who published a work on the approximation of the ratio of the circumference to the diameter of a circle, and *Albert Girard* a Fleming, who first gave the rule for determining the area of spherical triangles and polygons, and for measuring and comparing solid angles. *Kepler* was born in 1571. He introduced a new principle in geometrical investigation and in his *Nova Stereometria* presented for the first time the doctrine of *infinites*. In this tract, which was intended to give general rules for measuring the capacities of casks and vessels, he regards a circle as composed of an infinite number of indefinitely small triangles, having their vertices at the centre, and their bases at the circumference, and in the same manner are cones supposed to consist of an infinite number of small pyramids. By this happy conception, Kepler was enabled to go far beyond the *method of exhaustions* of Archimedes, not only in facilitating the solution of the problems which were presented by the latter, but in making the rules more general. The doctrine of *infinites* here introduced, may be regarded as the foundation of the present science of the Differential Calculus. In 1635, Cavalleri published his work on *Indivisibles*, in which the views of Kepler were in some measure extended. He conceived a line as composed of an infinite number of points, a surface of an infinite number of lines, and a solid of an infinite number of surfaces, which elements of magnitude he called *indivisibles*. The rule for summing an infinite series in arithmetical progression had been long known, and the application of it to find the area of a triangle by the method of indivisibles, was easy. The next step was supposing a series of lines in arithmetical progression, and squares to be described on each of them, to determine the ratio of the sum of the squares to the greatest square

taken as many times as there are terms in the progression, Cavalleri showed that when the number of terms was infinitely great, the first of these sums is just one third of the second. This led to the cubature of many solids. Its application to the cone was direct. The cone containing an infinite number of circles, whose diameters diminished in an arithmetical progression, the areas of these circles would be to each other as the squares on the diameters. By his rule the sum of all the squares is *one third* of the greatest square multiplied by the number of terms; hence, the cone is one third of the cylinder with the same base and altitude.

In strictness, lines, however multiplied can never make an area, or any thing else but a line, nor can areas, however added together make a solid. Still the conclusions of Cavalleri are true, though based upon an opposite supposition. This results from the consideration that Cavalleri regarded the ratio of the series used in the investigation and not the areas themselves, in defining the solidity of geometrical magnitudes. Thus, though the supposition that a certain series of lines may compose an area, be untrue, and also, that another series may compose another area—still it is true, that one of these areas must bear to another, the same ratio, which the sum of the one series of lines bears to the sum of the other series. We find this principle similarly applied in the Differential Calculus, in which the changes which variable quantities undergo, are determined by means of the ratios of these changes.

About the same time with Cavalleri lived, *Fermat*, a native of France, who added many new discoveries to the science of Geometry. Archimedes had determined the area of the common parabola, but Fermat extended the solution to embrace parabolas of all kinds. He was the author of the method of maxima and minima.

Roberval another French geometer, though inferior to Fermat, wrote a treatise on indivisibles, which was not published until after his death in 1693. He followed out the suggestions of Cavalleri and solved many curious

problems. The area of the cycloid had been hitherto an impossible problem. It had been tried by the best mathematicians of former times, but without success, when in 1634 Roberval demonstrated its area to be three times that of the generating circle. The introduction of the science of Analytical Geometry directed the attention of the great mathematicians of this time to the examination of a science which was about to effect a complete revolution in the methods of geometrical investigation. To *Descartes* a cotemporary of Fermat and Roberval, is due the credit of this new discovery, a more particular examination of which will be given in its proper place. Much rivalry existed between these three mathematicians. On the discovery of the area of the cycloid by Roberval, a copy of it was sent to Descartes, who returned it with a short demonstration of his own. Roberval intimated that Descartes had been aided in his solution, by the knowledge that it had been done. Descartes found out the tangents to the curve and challenged Fermat and Roberval to do the same. Fermat succeeded, but Roberval failed. In this problem Descartes made use of the doctrine of infinites as introduced by Kepler. Montucla calls this curve the *Helen* of geometers. Its *beauty* was the cause of war and produced long controversies.

Huygens who was born in 1629 and *Dr. Barrow* in 1630 were both distinguished mathematicians. The former solved many interesting problems, and showed how the problem for the rectification of a curve might be reduced to that of its quadrature. Barrow was the tutor of Newton.

After this period the cultivation of Geometry as a science became more general, and many names are recorded in England and France of great distinction on this subject. Among the English may be enumerated *Gregory*, *Simson*, *Stewart*, *Horsley*, *Playfair*, *Bonnycastle* and *Leslie*, and *Tacquet*, *Monge*, *Dupin Vallee*, *L'Huillier*, *Carnot*, *La Croix* and *Le Gendre* among the French.

About the year 1790, *Gaspard Monge* introduced in-

to France a new species of Geometry, called Descriptive Geometry. Its object was two-fold. 1. The accurate representation on *planes* of all geometrical magnitudes, and secondly, the construction of graphic problems involving three dimensions. Some hints had for many years been thrown out, by architects, who needed this species of representation, showing how these constructions might be determined. *Courcier*, a jesuit, published a work, in 1663, showing how to describe the intersections of cylindrical, conical and spherical surfaces. But Monge was the first to collect these detached principles into a science, to which he gave the name of Descriptive Geometry, and in which he included practical problems in stone cutting, carpentry, shades and shadows and perspective. Monge's work was republished in 1812 by *Hachette*, with a supplement, the subject having in the mean time been made a necessary part of the course in the Normal and Polytechnic schools of France. In 1818 *Vallee's* treatise appeared, and the science soon met with very general adoption throughout Europe. In this country but two works have appeared on the subject, one by *Col. C. Crozet* while Professor of Mathematics at West Point, and the other by *Professor Davis*, which was based upon Crozet's, with a supplement on the warped surfaces from the able work of Vallee.

ALGEBRA dates a more recent origin than either of the two sciences we have been just considering. It was not until Arithmetic had advanced to a considerable degree of perfection, and mathematicians had commenced to feel the necessity for abridging, as well as generalizing, its operations, that Algebra was introduced. In the early part of the 13th century, *Leonardo*, a merchant of Pisa, having made repeated visits to Arabia, returned to Italy with a knowledge of Algebra. A manuscript of his is quoted as far back as 1202. Though this important science was introduced into Europe from Arabia, it is by no means certain that Arabia is its native country. There is reason to believe that it is probably due to the Hindoos. Traces of this science are to be found among this

ancient people bearing date about 950. By the manuscript of Leonardo it appears that his knowledge of Algebra did not extend beyond quadratic equations. The language is very imperfect, corresponding to the infancy of the science, the quantities and the operations being expressed in words, with the aid of a few abbreviations. The rule for solving quadratic equations by completing the square, is demonstrated geometrically. This manuscript still remains and has never been published. The first printed Algebra is that of *Lucas de Burgo* in 1494. It was in Italian. He calls Algebra, *l'arte maggiore*. The known number is called *no.* or *numero*, and *co* or *cosa* stands for the unknown quantity.—Hence Algebra was called the *cossic art*. The square is represented by *ce*, the cube by *cu*, and *p* and *m* stand for *plus* and *minus*. Thus 3 co, p 4 ce, m 5 cu, p 2 ce. ce. m 6 n°. represented $3x+4x^2-5x^3+2x^4-6$. Thus the first appearance of Algebra is that of a system of *short-hand writing*, or abbreviation of common language applied to the solution of arithmetical questions. It was a contrivance simply to save trouble, and yet to this contrivance we are indebted for the most philosophical and refined art which has ever been employed to convey ideas. This scientific language, like every other, has advanced to perfection by degrees, and from a rude and weak beginning.

The next century after the publication of Lucas de Burgo's Algebra, a work was discovered written by *Diophantus* of Alexandria about the 3rd century, which though treating of arithmetical questions, solves them in a manner, that may be regarded as algebraic. The expression is that of common language, abbreviated and assisted by a few symbols. This method is still called the Diophantine analysis, and consisted in transforming irrational numbers into squares and cubes.

The name of *Jerome Cardan* stands conspicuous in the history of Algebra. He was born at Milan in 1501. He was a man of great talents and industry, but professed divination and died to fulfil an astrological prediction.

Before his time very little advance had been made in the solution of equations of a higher degree than the second, except that in 1508, *Scipio Ferrari* of Bologna, is said to have discovered a rule for solving a single case of cubic equations, viz $x^3+px=q$. He communicated the secret to one of his pupils, who, as was the custom in those days, challenged *Nicholas Tartaglia* to contest with him in the solution of algebraic problems. Tartaglia had in the mean time found out the rules for two other cases $x^3+px^2=q$ and $x^3-px^2=q$, and when the contest arrived, he was not only able to solve the problems offered by his antagonists, but to baffle them by others, the solution of which he had discovered. This was in 1535, and 4 years afterwards, Cardan obtained the secret from Tartaglia, under an oath of secrecy. In 1545 Cardan published his *Ars-magna*, in which he gave the rules of Tartaglia, violating his oath, but giving the credit to Tartaglia. This rule which bears the name of Cardan, consists in substituting for the unknown quantity a new unknown quantity connected with some arbitrary constant, to which such a value is attributed as shall cause the second term of the cubic equation to disappear. The solution of a complex cubic equation is thus reduced to that of a simple form, viz $x^3=b$. This breach of faith on the part of Cardan, having produced an enmity between him and Tartaglia, the latter defied Cardan to a contest, in which each should prepare 31 problems to be solved by the other. Cardan accepted the challenge and gave a list of his problems to the other, but devolved the duty of meeting his antagonist upon his pupil *Ferrari*. The problems of Tartaglia were so much more difficult than those of Cardan, and Ferrari so frequently failed, as to place Tartaglia in a high rank among algebraists, although he made but few discoveries. Cardan was acquainted with negative and positive roots, the former of which he called *false* roots, and was also familiar with many of the general principles of equations. Cardan's rules are all given in verse, to facilitate remembering them, an expedient entirely disused by the substitution of formu-

læ, which are more readily and permanently impressed upon the mind, than by any metrical combination of words.

Besides in Italy, the science of Algebra was making rapid progress in Germany, France and England. In 1544, *Stifelius*, a German, published a book on algebra, in which were introduced for the first time the signs for *plus* and *minus*. *Robert Recorde* a little later published the first English treatise on this science, called by the singular title " *Whetstone of Wit.*" In this was introduced the present sign of equality.

Pelitarius, a French mathematician, in a treatise dated 1558, was the first who showed that the root of an equation is a divisor of the last term. *Bombelli*, an Italian, about the same time, published a work, in which he considered the irreducible case of Cardan's rule, and was the first to remark that such problems could always be solved by the trisection of an arc. But the greatest boast of France and algebraic science, was *Francis Vieta.* He was the first to use letters to denote known as well as unknown quantities. He cleared equations of their co-efficients, obtained a new solution for cubic equations different from Tartaglia's and discovered the relation between the roots of an equation, and the co-efficients of its terms, in the case in which none of the terms are wanting. To *Vieta* is due the credit of making the first step in the application of Algebra to Geometry. The problems to which the investigations of Victa were directed were those of determinate geometry only. He succeeded in showing how algebraic expressions might be represented by geometrical constructions, so that the values of the unknown quantity in the given equation might be interpreted. This important step led the way to the more enlarged and triumphant discovery of Descartes, which constitutes the science of Analytical Geometry.

A valuable treatise on algebra was published by *Albert Girard* in 1669. This author extended the truth partially discovered by Vieta, and demonstrated the successive formation of the co-efficients of an equation, from the com-

bination of the sum of its roots, the sum of the products of the roots, taken two and two, three and three, &c. He appears to have been the first who understood the use of negative roots, and called them quantities less than nothing. He also showed that the number of roots in any equation could not exceed its degree.

The person next in order as a contributor to this science is *Thomas Harriott*, an English Mathematician, whose treatise on Algebra was published after his death in the latter part of the 17th century. He was the friend and companion of Sir Walter Raleigh in his second expedition to Virginia, in whose house he spent the latter portion of his life. He perfected the general theory of equations, explaining the truth in its fullest extent, to which Vieta and Girard had been approximating. He used the smaller instead of the larger letters of the alphabet, by which improvement, simple as it was, the form of algebraic language was brought nearer to that of the present day.

It will thus be seen by what gradual steps, the most eminent Mathematicians arrived at a full knowledge of the properties of equations. Without going wrong their progress was slow and laborious in developing truths, which when known seem neither difficult nor abstruse.

This peculiarity, which indeed characterizes all the mathematical sciences, places them in remarkable contrast to all others. By how many various and contradictory suppositions, was the "system of the world" attempted to be explained, before the discovery of the law of universal gravitation by Newton? Whether the sun moved around the earth, or the earth around the sun, was a question, whose solution before the time of Copernicus, fully justified the reply of the schoolboy to his teacher in later days, when he said, "they took it turn and turn about." What is here said of Astronomy is in a great measure applicable to all the other departments of physical science. The *truth* is not always reached by a gradual developement of dependent and less important

truths, but error and truth are so often found combined, that in many cases chance or well conceived hypotheses have led to the great discoveries which adorn these branches of human knowledge. I mention this with no design of underrating the physical sciences, but with the view of presenting a marked difference between them and those we are now considering. And this difference is easily explained. The mathematical sciences are based upon principles which are not only *true* but *immutable*, and the investigator may make an unlimited developement of these principles according to his labor and ingenuity, without ever going wrong. In determining, on the other hand, the laws which govern the physical world, we meet at the very threshold of the enquiry a difficulty which finite minds might readily anticipate in judging of the works of an infinite First Cause; and we have to resort to observation and hypotheses, until such theories are formed, as will satisfactorily account for, and reconcile the various phenomena of nature.

The succession of discoveries just mentioned, brought algebraic analysis to a state of perfection, very little short of what it is at the present day, and prepared it for the step which was about to be taken by *Descartes*, and which forms one of the most important epochs in the history of mathematics. The use of letters or numbers to represent lines had been long known, as has been mentioned, to Vieta, by whom this principle had been applied to the construction of problems of determinate geometry. Descartes extended the principle of Vieta so as to embrace any problem of indeterminate geometry. He referred a point in a plane to two lines whose position is supposed to be known, as axes, the point being determined when its distances from these lines were known. Calling these distances the *co-ordinates* of the point, the position of the point would vary as its co-ordinates varied. If now a point move in a plane in such a manner that there shall exist a constant relation between its co-ordinates, it follows, that this relation may be expressed by an equation, which will serve to define the curve gen-

erated by the point. For example, suppose the relation between the co-ordinates be such, that they shall be equal to each other for every position of the moving point, calling x and y these co-ordinates, the equation $x=y$ would express the equation of the line, which could be none other than a right line making an angle of 45° with the axis of x. Generalizing this result, Descartes assumed the possibility of expressing every curve by means of an equation between the co-ordinates of all of its points, which equation would serve to define the curve as perfectly as it could be by any artifice imaginable. Operating afterwards upon this equation by the known rules of algebra, the character of the curve would be ascertained, its extent determined, and its fundamental properties developed. The application of algebra to geometrical problems would no longer depend upon the skill and ingenuity of the investigator. The sole difficulty would consist in solving the equation representing the curve, for, as soon as its roots are known, the number and extent of the branches of the curve would be readily determined. Descartes was but 20 years of age when he made this great discovery, and his work on the subject containing 106 quarto pages was published in 1637.

One of the greatest ornaments to mathematical science in the 17th century was *John Napier* laird of Merchiston. With a view af shortening the tedious operations of multiplication and division required in the construction of trigonometrical tables, Napier was led to the discovery of *Logarithms*. His discovery was first published in Edinburg in 1614, and is one of the most remarkable instances of sagacity in the history of mankind. What adds to its peculiarity is the fact, that it came *complete* from the mind of its author, having received no improvement since his time. It has been supposed that an arithmetical fact mentioned by Archimides put Napier in the course of his discovery. It is this, that in a geometrical series, the sum of the indices of any terms of the series, is the index of the product of these terms. This is evi-

dent in algebra, since products are obtained by addition of indices. It was a necessary consequence, that if all numbers could be treated as terms of a progression, and their indices found like those of an ordinary progression, the method of finding the products of terms by the addition of indices would be universal. Napier first proved the possibility of this, and demonstrated the principle, and explained the manner of intercalating the terms when the numbers given were not in geometrical progression. In the original tables of Napier the logarithm of 10 was 3.0225850, the base being 2.7182818 and the modulus 1. At the suggestion of *Henry Briggs* of Gresham College, who was an able coadjutor of Napier, the base was afterwards changed to 10, the modulus of which is 0.434294482. The latter system is the one in common use, and is called the *common* system of logarithms. The *Naperian* or *hyperbolic* system as the other is called, is also used, the simple value of its modulus facilitating in a great measure the logarithmic calculus. Since the tables of Briggs published in 1624, various logarithmic tables of the trigonometrical lines, as well as numbers generally, have been published, the principal of which are those of Vlacq in 1633, Scherwin in 1724, Gardiner in 1741, Aubert in 1770 and Callet in 1783. The tables of Callet have been frequently re-publishsd and are very full and complete.

Besides the discovery of Logarithms, Napier contributed largely to the science of Trigonometry, by the addition of two theorems for the solution of spherical triangles. By the first he proved that in every right angled spherical triangle, radius multiplied by the sine of the middle part, was equal to the rectangle of the tangents of the adjacent parts, or the rectangle of the cosines of the opposite pars. By Napier's analogies, having given the two sides and the included angle, or two angles and the included side of a spherical triangle, the remaining parts are determined.

To the same century, though a little later belongs the name which by general consent has been placed at the

head of those great men who have been the ornaments and benefactors of their species. Whether we consider his character as a philosopher, mathematician or Christian, the name of *Sir Isaac Newton* stands pre-eminent. He was born at Woolsthorpe, in Lincolnshire England, Dec. 25, 1642. Evincing a decided taste for mathematical pursuits, he was at an early age placed under the tuition of the celebrated Dr Barrow, at that time professor of Mathematics in Trinity College, Cambridge. He soon became familiar with the works of Euclid and the geometry of Descartes, and when about 21 years of age, commenced the study of the works of Dr Wallis, the savillian professor of Geometry at Oxford. His treatise entitled *Arithmetica infinitorum* particularly delighted Newton. Dr Wallis had observed, that if the equations of curves of which he had given the quadrature, were arranged in a series, their areas would form another series. He saw also that the equation of the circle was intermediate between the first and second terms of the first series, and he concluded that by interpolating a term between the corresponding terms of the second series, he would have the area of the circle. Dr Wallis did not succeed in obtaining the indefinite quadrature of the circle, because he did not use general exponents, but he expressed the entire area by a fraction, the terms of which were obtained by the continued multiplication of a certain series of numbers. With the view of seeing how he could interpolate the terms for the second series of Dr Wallis, Sir Isaac Newton investigated the arithmetical law of the co-efficients of the series, and obtained a general method of interpolating, not only to the series in question, but all others also. Following out this idea he arrives at a general method of developing radical expressions composed of several terms into a series, and was thus led to the discovery of the celebrated *Binomial Theorem.* After applying this theorem to the rectification of curves, he discovered the general principle of deducing the area of curves from the ordinate, by considering the area as a nascent quantity, increasing by a con-

tinual fluxion, in the proportion of the length of the ordinate, and supposing the abscissa to increase uniformly with the time. Imitating Cavalleri, whose steps towards this discovery have been mentioned, he regards lines as generated by the motion of points, surfaces by the motion of lines &c. and by considering that the ordinates and abscissas of curves vary according to a regular law depending upon the equation of the curve, he deduced from this equation the velocities with which these quantities are generated, and by the Binomial theorem obtained the ultimate value of the quantity required. To the velocities with which every quantity is generated, he gave the name of *Fluxions*, the quantities themselves being called *Fluents*. This method constitutes the doctrine of the Differential and Integral Calculus. This discovery is supposed to have been made prior to 1666.

In the first edition of his Principia which was published in 1687, Newton made known the fundamental principle of the Fluxionary calculus. No information was however given of its notation, and it was not until 1693, that this was communicated to the world, in the 2nd volume of Dr. Wallis' works. In 1707 Mr. Whiston published the algebraic lectures of Newton while at Cambridge, under the title of *Arithmetica Universalis.* An English translation was soon after published by Mr. Ralphson, and a second edition with the author's improvements, was published at London in 1712 by Dr. Machin, Secretary to the Royal Society. Several minor works and tracts were published at various times but no mention was made of his great discovery of the Fluxinary Calculus. It is impossible to say what reasons operated with Newton in concealing this discovery, a neglect which was near depriving him of the credit to which he seems so justly entitled. Leibnitz a German mathematician, visited London in 1673 and became acquainted with the great men who then adorned the British Capitol. Returning shortly after to Paris, he devoted himself to the study of the higher mathematics, being aided therein by the talents and labours of Huygens. In

1677 Leibnitz communicated to Newton a new method of drawing tangents, at the same time explaining the principles of his calculus. He describes the algorithm, and the formation of differential equations, and the applications of the calculus to various geometrical and analytical questions. This was none other than the Differential calculus, which had been for years known to Newton. In 1684 Leibnitz published a full account of his discovery, and showed the method of determining the maxima and minima of curves—so that while the science of the Differential calculus was making rapid advances on the continent, under the direction of Leibnitz and the two Bernouille, Newton had not published a word on the subject. The silence of Newton was at last broken, and in the second book of his Principia, he explained the fundamental principles of his fluxionary calculus.

The importance of the discovery of this new calculus to the cause of science, and the celebrity of the two rival mathematicians who had separately produced it, led to an animated contest between them, which was for a time pursued with great bitterness and violence on the part of Leibnitz and his friends. The result of the whole was the conviction that Newton invented fluxions at least ten years before Leibnitz. Newton was therefore the *first* inventor and Leibnitz the *second*. The talents which Leibnitz displayed in improving the calculus, showed that he was capable of inventing it, and his character stood sufficiently high to repel every suspicion of plagiarism.

In 1701 Newton was appointed President of the Royal Society, and in 1705 was knighted by Queen Anne. He died March 20, 1727, aged 85. What adds to the interest felt in the life of this illustrious man, is the fact, that he was a firm believer in the great doctrines of the Christian faith. He was not only distinguished by an external respect for religion, but he was a *Christian* from his youth, and was in the daily habit of reading and studying the Holy Scriptures.

All the departments of algebraic analysis underwent a rapid

improvement on the introduction of the science of the Differential calculus. In France, especially, were genius and industry combined for its promotion, the chief contributors to which were *La-Grange*, *Garnier*, *Francœur*, *Carnot*, *Cousin*, *La-Croix*, *Bourdon*, *Biot*, *Boucharlat*, *Bezout*, *Reynaud*, *Monge*, *Puissant*, whose works are at this time the text books in the higher schools of France and Europe, the translations of many of them being also adopted in England and this country.

The History of this country extends over too limited a space, to lead us to expect much addition to the cause of mathematical science during so brief a period. The peace of '83 found the United States possessing few men devoted to scientific pursuits, and fewer institutions affording facilities for the prosecution of such studies. At that time there were but 8 colleges in existence, and these but so imperfectly organized, that instruction in the sciences was more in name than in fact. Upon the organization of the U. S. Military Academy at West Point, in 1802, a new spirit was impressed throughout the country, and the number of colleges rapidly increased, but it was not until 1817 that this institution became so conspicuous in the assistance it rendered the cause of science. The President of the Board of Visitors of the Virginia Military Institute, *Colonel Claude Crozet*, a native of France, was the first to introduce into this country, the improvements in Mathematical science, which the developements of the past century had made known to Europe. While Professor of Mathematics at West Point, he introduced the use of the *Black Board* now become so common and necessary an instrument for imparting instruction. He also gave instruction in Analytical Geometry and Descriptive Geometry which were, hitherto, almost unknown in the country. He published a small treatise on the Conic Sections and Descriptive Geometry, which displayed much originality of thought. He added much to the course of Civil Engineering and introduced the Topographical signs which are now used in military drawing.

In 1826 Professor Davies commenced a series of text books for the use of the Cadets, all of which were translations or compilations from the French. He was educated at West Point and his Text Books enjoy a deservedly high reputation. His Algebra is from the translation of the admirable work of Bourdon, by *Professor E. C. Ross*, formerly of West Point, now of Kenyon College. *Nathaniel Bowditch* of Massachusetts is a name of which Americans may proudly boast, for his labors in and contributions to, the cause of Mathematical Science. His Navigator has received a circulation unequalled by any work of the kind in existence. But the work which has added most to the reputation of Dr. Bowditch is his translation and annotation of the illustrious treatise of La Place, the *Mecanique Celeste*. Dr. Bowditch was severally invited to the chair of Mathematics at West Point, and the University of Virginia, but he remained to the close of his life in the superintendence of one of the Marine Insurance Companies in Boston.*

The establishment of the *Virginia Military Institute* may be regarded as another prominent step towards the advancement of Mathematical Science. Based as it is, upon the West Point Academy, it is natural to suppose that these subjects will claim a high consideration in its course of instruction. Its operation has, however, been too short to say what its effects have been, or what they will hereafter be. A band of young and chivalric *Virginians* armed in the noble work of advancing the cause of Science, can do much; and I look forward with much certainty and satisfaction, to the period, when Virginia, which now boasts so long a line of Statesmen and Orators, will have no less cause to glory in the number and greatness of her Sons of Science.

*It would have been a pleasing duty to add in this connexion the names and services of other distinguished Americans, who have by their writings contributed to the promotion of the Mathematical Sciences. The want of authentic information alone forbids a mention of them at this time.

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